CHAPTER

3 Linear Equations

At this point, you might think about how you and your student are interacting. For example, are you being a student to your student? Do you explain as little as possible or just enough so that your student is becoming an independent learner and thinker? Is the pencil or calculator in your student's hands? Do you try not to answer questions that your student hasn't asked? Telling your student too much can waste time, because your student might not understand. It can lead to your student feeling overwhelmed. Deeper understanding *can* result when you allow your student to teach the concept or skill to you and others.

Content Summary

Chapter 3 focuses on equations of lines. Students expand their idea of linearity, and they learn to work backward.

Linearity

You can think of linearity in several ways. *Linearity as constant rate of change.* One way to think about linearity is that the rate of change of one variable in relation to the other is constant. You start somewhere and advance by the same amount at each step. This kind of change is called *linear growth,* although the values will be shrinking instead of growing when the rate of change is negative. With variation in Chapter 2, growth always started at 0; in this chapter, growth can start with any value.

Linearity as equations of the form $y = a + bx$*.* Another way of thinking about linearity is through equations that relate variables. In this book, linear equations have the *intercept form* $y = a + bx$*.* This form indicates the starting point, a, and what's added to it, b, each time x increases by one unit. The traditional *slope-intercept form* $y = mx + b$ that you might remember is mentioned in Chapter 4, but the intercept form introduced in this chapter better reflects the idea of growing at a constant rate from a starting point.

Linearity as graphs of lines. You can also understand linearity through graphs. The equation $y = a + bx$ is graphed by starting at point $(0, a)$ and moving vertically by amount b for each unit moved across from left to right.

Working Backward

Many real-life situations call for predicting when a quantity will grow to a certain value. Ways of making that prediction reflect the three ways of thinking about linearity. From the growth perspective, you might think of counting the steps as you repeatedly add on to the starting point until you reach the desired value. This can be done by hand, or you can use home-screen recursion or sequences on a graphing calculator.

If the situation is represented by an equation, there might be two methods of solving it: the *undoing* method and the *balancing* method. If you know that $3x + 2 = 17$, you can use the undoing method and think, "I multiply x by 3 and add 2 to get 17. To find x, I can undo that process, beginning with 17. I subtract 2 (to get 15) and then divide by 3 (to get 5)." You'll need the balancing method if the unknown appears more than once. Applying the balancing method to the equation $3x + 2 = 17$, you subtract 2 from both sides to get $3x = 15$, and then divide both sides by 3 to get $x = 5$.

(continued)

Summary Problem

You and your student might discuss this summary problem from Chapter 3. It's a good problem to revisit several times while working through the chapter.

Here's a table showing the heights above and below ground of different floor levels in a 25-story building (taken from page 158):

Floor number	Basement (0)	1	2	3	4	…	10	…		…	25
Height (ft)	-4	9	22	35		…		…	217	…	

What floor has a height of 282 feet?

Questions you might ask in your role as student to your student include:

- How far apart are the floors?
- What could the negative number mean?
- Could you solve this problem by recursion on a graphing calculator?
- Is it possible to represent the height by an algebraic expression?
- Does the distance between floors appear in the expression?
- Does the height of the basement appear in the expression?
- Can the whole problem be represented by an equation?
- Can you graph the equation?
- What are various ways of solving the equation?
- Could you make an equation that tells how long it takes an elevator to reach various floors?
- In the Empire State Building in New York City, the floors vary in heights. Could you still write an equation that might be useful for either the heights or the elevator time?

Sample Answer

The floors, which start with a negative number (possibly meaning that the basement floor is below ground level), are 13 feet apart. To solve the problem on a graphing calculator, you could start with -4 and repeatedly add 13 until you reach 282. It would be most efficient to use recursion on a list [see **Calculator Note 3A**] to keep track of both the floor number and the height.

Or you can solve the equation $\textit{height} = -4 + 13 \cdot (\textit{floor number})$, or $282 = -4 + 13x$. Using the balancing or the undoing method, you can solve the equation to get $x = 22$. Ask what 22 represents (the floor with a height of 282 feet). If you know the time it takes for the elevator to travel one floor, you can use that number in place of 13 to find the time it takes the elevator to travel from the basement to any other floor.

For buildings with irregular floor heights, you might use an equation containing the *average* height to make estimates of height or time. Or you might use different equations for different parts of the building.

Chapter 3 • Review Exercises

Name ____________________ Period __________ Date ______________

1. *(Lessons 3.1, 3.2)* Plot the first six points represented by each recursive routine.

a. $\{-4, 2\}$ ENTER

$\{\text{Ans}(1) + 1, \text{Ans}(2) + 3\}$ ENTER

b. $\{0, 1.5\}$ ENTER

$\{\text{Ans}(1) + 1, \text{Ans}(2) - 0.25\}$ ENTER

c. $\{2, -2\}$ ENTER

$\{\text{Ans}(1) + 1, \text{Ans}(2) + 0.5\}$ ENTER

2. *(Lessons 3.3, 3.4)* The table at right shows a person's distance from a motion sensor at various times.

Time (s)	Distance (m)
0	0.3
1	0.7
2	1.1
3	1.5
4	1.9

a. Describe the walk shown in the table. Include where the walker started, how quickly the walker walked, and in what direction the walker walked.

b. Write a linear equation for the walk, in intercept form. Graph the equation and plot the points from the table.

3. *(Lessons 3.5, 3.6)* A local theater company has a yearly membership fee, and members pay a reduced per-ticket cost. The equation $C = 25 + 8n$ expresses the total cost C of purchasing n tickets in a single year.

a. Based on the equation above, what is the yearly membership fee? What is the cost per ticket?

b. If a person does not want to buy a membership, theater tickets cost \$10 each. Write an equation for the total cost C of purchasing n tickets for someone without a membership.

c. Graph both equations for the cost of n tickets. What is the rate of change of the cost for a member? For a non-member?

d. Christina looked at the schedule for the upcoming theater year, and she found 12 shows that she would like to attend. Should she buy a membership?

e. Last year, Christina bought a membership, and she spent a total of \$137 that year on the membership fee and theater tickets. How many tickets did she buy?

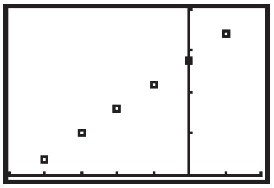
4. *(Lesson 3.6)* Give the additive inverse of each number.

a. 1 **b.** -1.25 **c.** $\frac{3}{4}$ **d.** $-\frac{6}{5}$

5. *(Lesson 3.6)* Give the multiplicative inverse of each number in Exercise 4.

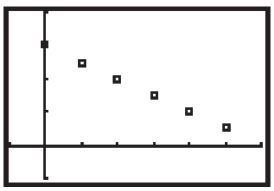
SOLUTIONS TO CHAPTER 3 REVIEW EXERCISES

1. a. The graph should include the points $(-4, 2)$, $(-3, 5)$, $(-2, 8)$, $(-1, 11)$, $(0, 14)$, and $(1, 17)$.



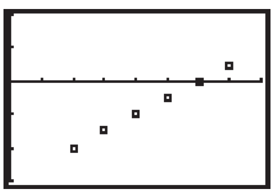
$[-5, 2, 1, 0, 20, 5]$

b. The graph should include the points $(0, 1.5)$, $(1, 1.25)$, $(2, 1)$, $(3, 0.75)$, $(4, 0.5)$, and $(5, 0.25)$.



$[-1, 6, 1, -0.5, 2, 0.5]$

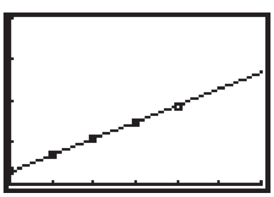
c. The graph should include the points $(2, -2)$, $(3, -1.5)$, $(4, -1)$, $(5, -0.5)$, $(6, 0)$, and $(7, 0.5)$.



$[0, 8, 1, -3, 2, 1]$

2. a. The walker started 0.3 m away from the sensor, and walked away from the sensor at a rate of 0.4 m/s.

b. $y = 0.3 + 0.4x$

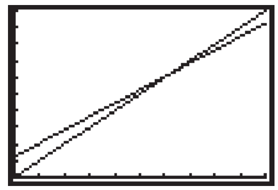


$[0, 6, 1, 0, 4, 1]$

3. a. \$25; \$8. The total cost is (*membership fee*) + (*cost per ticket*) · (*number of tickets*).

b. $C = 10n$

c. Graph $y = 25 + 8x$ and $y = 10x$; Rate of change for member: \$8 per ticket; for non-member: \$10 per ticket.



$[0, 20, 2, 0, 200, 20]$

d. Christina should not buy a membership. Compare the total cost under each option. With membership, $C = 25 + 8 \cdot 12 = 25 + 96 = 121$; without membership, $C = 10 \cdot 12 = 120$. She will save \$1 by not getting a membership.

e. 14 tickets. Solve the equation $25 + 8n = 137$. This solution uses the balancing method; students might also solve by undoing.

$25 + 8n = 137$	Original equation.
$25 + 8n - 25 = 137 - 25$	Subtract 25 from both sides.
$8n = 112$	Combine like terms.
$n = 14$	Divide both sides by 8.

4. Take the opposite of each number. The additive inverse is the number that is added to the given number to equal 0.

a. -1 **b.** 1.25 **c.** $-\frac{3}{4}$ **d.** $\frac{6}{5}$

5. Find the reciprocal of each number. The multiplicative inverse is the number that is multiplied by the given number to equal 1.

a. 1 **b.** -0.8 **c.** $\frac{4}{3}$ **d.** $-\frac{5}{6}$